Technical Correction to the First Printing

Overview and Implications

In appendix 1B, equation B.5 (p. 53) relating the inequality parameter "σ" of the lognormal distribution to the Gini coefficient contains an error, as examined below.[1] This parameter is important in estimating the elasticity of poverty with respect to growth. This elasticity, in turn, affects the estimates of the impact of trade liberalization on global poverty. It turns out that the correction of the error causes a moderate reduction in the estimate for the long-term reduction of global poverty from complete free trade, from a central estimate of 540 million people lifted out of poverty to 440 million. The corresponding "high" estimate declines from 680 million to 580 million. The broad conclusion remains unchanged, that global free trade could lift somewhere in the vicinity of 500 million people out of poverty over the long term.

An important reason for the limited change in the estimates is that the original estimates imposed a ceiling of 3.5 on the poverty elasticity of growth. Most of the Asian countries had estimated elasticities higher than this ceiling, so the calculations of their poverty impacts were constrained by the ceiling elasticity. With the recalculation of the country poverty elasticities, the Asian countries still tend to have relatively high elasticities, in the range of 2.5 to 3 (table TC.1 in this note). The reductions in the calculated poverty effects for these countries are thus not as great as would have been the case if the original calculations had not been constrained by the ceiling elasticity permitted. This constraint, in turn, was adopted based on the judgment that the range of empirical estimates of the elasticity rarely exceeded 3.5.

Corrected values for the most important tables affected (tables 1.8, 1B.1, and 5.3) are presented at the end of this note. The range of the poverty elasticity in table 1B.1 (p. 54) is now from a high of 7.67 to a low of 0.58, instead of the original estimated range from 25.3 to 0.43. Summary table 5.3 (p. 252) combining static, dynamic productivity, and dynamic investment effects reports the change in the aggregate poverty effects just described and provides country detail. The specific revised values in tables 4.7 (static), 4.9 (steady-state), and 5.2 (dynamic productivity effects)—pages 210, 215, and 250, respectively—are not reported here but can be obtained by replacing the estimated or constrained country-specific poverty elasticities by the revised country-specific poverty elasticities reported here in table TC.1 and then applying the calculations of each of the respective tables

Finally, the "cross-section paradox" discussed in chapter 1 is also affected by the revision of σ. This paradox is that, for countries with per capita incomes above about $1,000, the lognormal predicted incidence of poverty is usually much lower than the observed incidence. After the cor-

[1] I am indebted to David Rosnick for calling the error to my attention. Note also that in equation B.1 (p. 52) the minus sign was inadvertently omitted from the exponent. Other errata: in table 4.1 (p. 180) "GDC" should be "DGC"; in table 4.2 (p. 185), "LDC" should be "DGC."

rection this diagnosis remains broadly true but is less extreme than before (revised table 1.8). The correction reduces the poverty elasticity for Gini coefficients below about 0.5 but raises it for higher Gini coefficients. As a result, for countries with lower Gini coefficients, the corrected "predicted" poverty incidence is higher than before, whereas predicted poverty for high-inequality countries is lower than before. The most extreme cases of divergence between predicted and actual poverty continue to show large gaps between the two, although the gaps are smaller than before. Thus, for China with actual poverty incidence at 53.7 percent, the original predicted level of only 0.5 percent is increased to 3.9 percent, leaving unchanged the qualitative diagnosis of a major gap.

Table TC.1 Original and corrected country-specific poverty elasticities used in calculation of poverty impacts

Country	Original	Revised	Country	Original	Revised
Bangladesh	3.5	2.3	Thailand	3.5	3.5
China	3.5	2.9	Argentina	3.1	2.9
India	3.5	2.5	Brazil	1.0	1.5
Indonesia	3.5	3.0	Mexico	2.0	2.1
Korea	3.5	3.5	Turkey	3.5	3.5
Malaysia	3.2	2.7	Mozambique	1.4	1.0
Pakistan	3.5	3.2	South Africa	1.2	1.7
Philippines	3.0	2.2	Tanzania	1.0	1.0
			Uganda	2.3	1.4

Corrected Relationship of the Lognormal Distribution Parameter to the Gini Coefficient[2]

Appendix 1B, p. 53, states incorrectly that:

$$\sigma = 2\left\{\pi^{-1}\left(\frac{G+1}{2}\right)\right\}^2 \qquad \text{(B.5)}$$

where σ is the standard deviation in the lognormal distribution, π is the standard normal distribution, and the negative exponent indicates inverse function. G is the familiar Gini coefficient obtained by taking the ratio of the area between the diagonal and the Lorenz curve to the full area under the diagonal (in the diagram of cumulative percent income, vertical axis, against cumulative percent households, horizontal axis).

This equation was derived from Bourguignon (2002), which was available at the time the original analysis was prepared (Cline 2002a).[3] In the subsequent, published version of the same paper (Bourguignon 2003), the relevant equation was corrected. The correct version of the underlying equation is:

$$G = 2\,\pi\left(\frac{\sigma}{\sqrt{2}}\right) - 1 \qquad (1)$$

[2] I am grateful to Aart Kraay for clarifications.

[3] In Bourguignon (2002), it was stated that: $G = 2\pi(\sigma/2)^{½} - 1$. The corrected published version (2003) stated that $G = 2\pi(\sigma/2^{½}) - 1$.

When this underlying relationship is rearranged and the inverse function applied, the corrected equation for σ becomes:

$$\sigma = \sqrt{2}\left\{\pi^{-1}\left(\frac{G+1}{2}\right)\right\} \qquad \text{(B.5')}$$

In other words, the initial equation for σ was instead the equation for σ^2. Dollar and Kraay (2001a, 12) also give equation (B.5') for σ.

This relationship between G and σ can also be confirmed from Aitchison and Brown (1963, 8; 12–13) in combination with Gastwirth (1972). Aitchison and Brown indicate that for the lognormal distribution, the Gini "mean difference" coefficient (pairwise absolute difference between all observations) is:

$$\hat{G} = 2\alpha\left\{2\pi\left(\frac{\sigma}{\sqrt{2}}\right) - 1\right\} \qquad (2)$$

where

$$\alpha = e^{\mu + \frac{1}{2}\sigma^2} \qquad (3)$$

In turn, Gastwirth (1972, 307) states that for any distribution function, the familiar Gini coefficient of relative income inequality (i.e., G) equals the absolute mean difference coefficient ($\hat{G}$) divided by $2m$, where m is the mean value of the distribution. However, because the value of Aitchison and Brown's α (equation 3) is simply the mean value of the lognormal distribution function, equation (2) for mean difference translates directly into equation (1)—that is, equation (2) divided by 2α becomes equation (1). Thus, Aitchison and Brown's value for the mean difference coefficient confirms the corrected value for σ shown in (revised) equation (B.5').

Corrected Tables

Revised table 1.8 Actual and predicted poverty rates for selected countries

Country	PPP per capita income (1990 dollars)	Gini coefficient (ratio)	Actual poverty incidence (percent)	Lognormal predicted poverty (percent)
Tanzania	500	0.38	59.7	81.3
Ethiopia	620	0.40	76.4	72.3
Nigeria	770	0.51	90.8	66.8
Senegal	1,400	0.41	67.8	31.8
India	2,230	0.38	86.2	10.7
China	3,550	0.40	53.7	3.9
Colombia	5,580	0.57	28.7	10.3
Brazil	6,840	0.60	17.4	9.9
Mexico	8,070	0.54	42.5	3.8

Revised table 1B.1 Lognormal poverty elasticity[a] as a function of Gini coefficient and ratio of mean income to poverty threshold income

		μ/y_p					
Gini	σ	**10**	**5**	**3.33**	**2.5**	**2**	**1.67**
0.3	0.54	7.7	5.5	4.2	3.4	2.8	2.3
0.35	0.64	5.5	4.0	3.1	2.5	2.1	1.8
0.4	0.74	4.1	3.0	2.3	1.9	1.6	1.4
0.45	0.85	3.1	2.3	1.8	1.5	1.3	1.1
0.5	0.95	2.4	1.8	1.4	1.2	1.0	0.9
0.55	1.07	1.9	1.4	1.1	1.0	0.8	0.7
0.6	1.19	1.5	1.1	0.9	0.8	0.7	0.6

a. Absolute value.

Revised table 5.3 Combined long-term static and dynamic effects of free trade on poverty
(millions, change from baseline)

	Static		Dynamic		Total	
Country	**Central**	**High[a]**	**Productivity effect**	**One-half additional net steady state effect[b]**	**Central**	**High**
Asia						
Bangladesh	–2.7	–12.0	–23.9	–2.4	–29.1	–38.3
China	–20.3	–82.7	–38.6	0.0	–58.9	–121.4
India	–20.1	–66.5	–52.9	–77.3	–150.3	–196.7
Indonesia	–5.2	–14.2	–7.6	–6.3	–19.1	–28.1
Korea	0.0	0.1	0.0	0.0	–0.1	0.0
Malaysia	–0.1	–0.1	–0.3	–0.5	–0.9	–0.9
Pakistan	–7.2	–12.3	–21.0	–42.3	–70.5	–75.6
Philippines	–1.7	–1.4	–2.2	–1.1	–5.0	–4.7
Thailand	–4.1	–4.0	–0.7	–9.1	–13.9	–13.8
Hong Kong, Taiwan, and Singapore	0.0	0.0	0.0	0.0	0.0	0.0
Other	–2.1	–8.8	–2.8	–5.9	–10.8	–17.5
Subtotal	–63.5	–202.1	–150.1	–144.9	–358.5	–497.1
Latin America						
Argentina	–0.7	0.2	–0.1	–0.1	–0.9	0.0
Brazil	–1.5	–0.9	0.1	–0.4	–1.9	–1.3
Central America and Caribbean	–3.2	–2.9	–1.1	–1.6	–5.9	–5.6
Mexico	–0.4	0.4	–0.2	–0.1	–0.8	0.0
Other	–2.7	–2.2	0.1	–3.3	–5.8	–5.4
Subtotal	–8.4	–5.5	–1.3	–5.5	–15.2	–12.3
Europe						
Central and Eastern Europe	–2.4	–1.0	–0.7	–0.2	–3.3	–1.9
Middle East and North Africa						
Turkey	–1.3	0.0	–0.6	–1.1	–3.0	–1.8
Other	–3.8	–4.4	–2.1	–6.4	–12.3	–12.9
Subtotal	–5.1	–4.4	–2.7	–7.5	–15.4	–14.7
Sub-Saharan Africa						
Mozambique	–0.5	–0.1	–0.2	–0.5	–1.2	–0.8
South Africa	–0.8	–1.3	0.1	–0.2	–0.9	–1.4
Tanzania	–1.4	–3.1	0.0	–0.5	–2.0	–3.6
Uganda	–0.4	–2.3	0.0	–0.4	–0.9	–2.7
Other	–15.6	–24.8	–0.8	–24.6	–41.0	–50.1
Subtotal	–18.8	–31.6	–1.0	–26.2	–46.0	–58.7
Total	–98.2	–244.6	–155.9	–184.3	–438.3	–584.7

a. Adjusts for chapter 3's agricultural sector estimates; see text.

b. One-half increment of steady state (induced capital investment) effects above static.

Source: Author's calculations.

Additional references

Bourguignon, Francois. 2003. The Growth Elasticity of Poverty Reduction: Explaining Heterogeneity across Countries and Time Periods. In *Inequality and Growth: Theory and Policy Implications*, ed., Theo S. Eicher and Stephen J. Turnovsky. Cambridge, MA: MIT Press.

Gastwirth, Joseph L. 1972. The Estimation of the Lorenz Curve and Gini Index. *The Review of Economics and Statistics* 54, no. 3 (August): 306–16.