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17-13 A New Index of External Debt Sustainability

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Abstract

Debt sustainability is fundamentally a probabilistic concept: Debt is rarely sustainable with probability one. We propose an index of external debt sustainability that reflects this uncertainty. Namely we construct the index as the probability that, at the current exchange rate, net external debt is equal to or less than the present value of net exports. Constructing this index involves three steps: (1) deriving the distribution of the present value of net exports at the current exchange rate; (2) deriving the distribution of exchange rates associated with the condition that, for each realization, the present discounted value of net exports is at least equal to the value of current net debt; and (3) assessing where the current exchange rate stands in the distribution of exchange rates and thus the probability that debt is sustainable. Having shown how this can be done, we then compute the index for two countries, the United States and Chile. Our main conclusion is the large degree of uncertainty implied by the presence of large gross asset and liability positions, together with uncertainty about rates of return on these assets and liabilities. The size of the distribution of exchange rate adjustments implies that one should be careful in concluding that debt is or is not sustainable at the current exchange rate and that strong measures are potentially needed to reestablish sustainability. Exchange rates that appear overvalued in the baseline may still imply a reasonably high probability that debt is sustainable at the current exchange rate; symmetrically, exchange rates that appear undervalued in the baseline may still come with a reasonably low probability that debt is unsustainable at the current exchange rate.

JEL Codes: F32, F34
Keywords: Sustainability, External Debt, Capital Gains, Gross Assets, Gross Liabilities, Uncertainty, Exchange Rates, Rates of Return, Valuation

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Debt is said to be sustainable if the borrower can service it now and in the future. In the case of the external debt of a country, this is often stated as: External debt is sustainable if net debt today is less than or equal to the present value of net exports.[1]

This definition however falls short of the mark in two ways:

- The first is that the present value of net exports is a random variable. For very bad outcomes, it is likely that the condition above will not be met. This implies that sustainability must be a probabilistic statement, for example, a high probability that the condition above will be met.

- The second is that net exports, as well as the value of net debt, depend on the exchange rate, and that, except in pathological cases (when a depreciation increases net debt more than it increases the present value of net exports), there is always an exchange rate depreciation that makes debt sustainable.

This suggests the following definition: External debt is sustainable if there is a high enough probability that, at the current exchange rate, net debt is equal to or less than the present value of net exports.

This is the definition we start from in this paper. Our goal is to construct a corresponding index of sustainability, defined as the probability that, at the current exchange rate, net debt is equal to or less than the present value of net exports.

To do so requires three steps:

- Deriving the distribution of the present value of net exports at the current exchange rate.

- Deriving the distribution of exchange rates associated with the condition that, for each realization, the present discounted value of net exports is at least equal to the value of current net debt;

- Assessing where the current exchange rate stands in the distribution of exchange rates, and thus the probability that debt is sustainable.

As will be clear, the construction of such an index raises a number of conceptual and empirical issues. Our approach to the various trade-offs is to propose an index that can be easily constructed based on existing data and available forecasts, and may for example be used by the IMF country desks in their assessment of a country's external position.

[1] So if the country is a net debtor, trade surpluses must be large enough. If the country is a net creditor, trade deficits must be small enough. Note that sustainability is a necessary but not a sufficient condition for optimality. For example, a positive net asset position together with a positive present value of trade surpluses imply that debt is sustainable, but not necessarily optimal.

The paper is organized as follows.

Section 1 goes through the basic derivation under the assumption of certainty and identical rates of return for assets and liabilities. Section 2 discusses a first complication, the fact that the relevant discount factor may sometimes be greater than one. Section 3 discusses a second complication, the fact that the rates of return on assets and liabilities can be quite different. Section 4 discusses how to introduce uncertainty about net exports and rates of return. Section 5 applies the methodology to the United States. Section 6 applies it to Chile. Section 7 concludes.

1 The basic derivation

Assume that external debt accumulation is given by:

$$D_{t+1} = (1 + r_t)D_t - NX_t \tag{1}$$

where D_t is net debt at the beginning of period t, NX_t is net exports in period t, and r_t is the real interest rate from t to to $t + 1$

Solving forward recursively gives:

$$D_t = \sum_{j=0}^{n} (\prod_{i=0}^{j} (1 + r_{t+i})^{-1}) \ NX_{t+j} + (\prod_{i=0}^{n} (1 + r_{t+i})^{-1}) \ D_{t+n+1} \tag{2}$$

Or, in terms of ratios to GDP:

$$d_t = \sum_{j=0}^{n} \prod_{i=0}^{j} \frac{1 + g_{t+i}}{1 + r_{t+i}} \ nx_{t+j} + \prod_{i=0}^{n} \frac{1 + g_{t+i}}{1 + r_{t+i}} \ d_{t+n+1} \tag{3}$$

where d denotes the ratio of external debt to GDP, nx denotes the ratio of net exports to GDP, and g denotes the rate of growth of GDP.

The condition that the ratio of debt to GDP does not explode implies that the last term must be non positive as n tends to ∞, so:

$$d_t \leq \sum_{j=0}^{\infty} \prod_{i=0}^{j} \frac{1 + g_{t+i}}{1 + r_{t+i}} \ nx_{t+j} \tag{4}$$

Sustainability requires that the ratio of net debt to GDP be equal to or less than the present value of the ratio of net exports to GDP. (For simplicity, we shall continue to refer in what follows to nx as "net exports" (rather than the ratio of net exports to GDP), and to d as "net debt" (rather than the ratio of net debt to GDP))

As of time t, future values of both nx, r and g are random variables, and so, by implication, is the present value of net exports. Suppose for the moment that we actually knew with certainty both future values of nx conditional on the exchange rate remaining at its current level, and future values of r and g. Then, the computation of sustainability would be straightforward. We would:

- Compute the present value of net exports. If it exceeded net debt, we would conclude that external debt was sustainable.
- If, at the current exchange rate, the present value was smaller than net debt, we would then compute the value of the exchange rate that made the equality hold. This would require computing the effect of the exchange rate on both net debt and net exports.[2]
- The answer would then be a simple one. If the inequality held at the current exchange rate, external debt would be sustainable. Otherwise it would not. If it did not, the computation would give us the adjustment of the exchange rate which would make it sustainable.

Under uncertainty however, we will have to compute the exchange rate associated with each potential realization, and thus obtain a distribution of exchange rates. The comparison will be not between two exchange rates, but between the current exchange rate and this distribution. Before we get there, we have however to deal with a number of complications, to which we now turn.

2 Discount Factors

A major issue in constructing the present value of net exports is the choice of the discount factor $(1+g)/(1+r)$. A simple computation will make the point. Using historical averages of annual growth rates and rates of return on liabilities for 24 countries over various periods (depending on data availability) yields a discount factor between 0.95 and 1.05, with 3 countries having a discount factor equal to or above one (see Appendix Table 1). Using instead rates of return on assets yields a discount factor between 0.96 and 1.09, with 9 countries having a discount factor equal or above 1.0. This raises both conceptual and empirical issues.

The conceptual issue is clear. A discount factor above 1 implies that debt is automatically sustainable: For any arbitrary level of net exports, net debt eventually reaches a constant value, positive if net exports are positive, negative otherwise.

[2] Note the analogy with the determination of the price level in the fiscal theory of the price level (Cochrane 2001). In that theory, sustainability of government debt requires that the price level be such that real government debt is less or equal to the present value of primary surpluses.

One may however conclude that a discount factor permanently above 1 is unlikely, so that sustainability is still potentially an issue.[3] What is clear however is that, even if below 1, the discount factor may be close to 1. This implies that net exports far in the future get a substantial weight in the present value computation. To the extent that we do not have such far ahead forecasts, and indeed have little confidence about the distant future, this raises an empirical issue.

Faced with these theoretical and empirical issues, we take the following short cut. We ask what exchange rate is needed to keep debt at some date in the future equal to or less than debt today. In terms of equation 3, we impose that $d_{t+n+1} = d_t$, so the condition becomes:

$$d_t \,(1 - \prod_{i=0}^{n} \frac{1+g_{t+i}}{1+r_{t+i}}) \;\leq\; \sum_{j=0}^{n} \prod_{i=0}^{j} \frac{1+g_{t+i}}{1+r_{t+i}} \; nx_{t+j} \tag{5}$$

Note that if nx, g, r were to be constant in the future, the exchange rate which satisfied equation (5) would be the same as that which satisfies equation (3). Otherwise, if for example a major change in net exports is likely to happen after time n, the two will differ. Also, our modified index will come with a tighter distribution of exchange rates than the index implied by (3), and this is an important warning about the interpretation of the index later. We see however no way to avoid such a shortcut.[4]

What should n be? Ideally it should be as high as we have informed forecasts for. The required forecasts are available from the World Economic Outlook (WEO)for up to 5 years ahead. By then, much of the cyclical movement in net exports is likely to be gone. This suggests using $n = 5$, and this is what we do later.

3 Different rates of return on assets and liabilities

We have derived the conditions above under the assumption that the rates of return on assets and liabilities were the same so we could just look at net debt. If however, the two rates are different, the composition of net debt between assets and liabilities matters, a point emphasized in a number of recent papers (Gourinchas and Rey 2013). And indeed, the evidence is that the two rates are often different. The United States typically pays a much lower rate on its liabilities than it receives on

[3] This sweeps under the rug a number of relevant issues, namely whether an average interest rate on debt lower than the average growth rate is sufficient to allow for Ponzi schemes, in which debt can be issued and never repaid. For a discussion of these issues in the context of domestic public debt, see Blanchard and Weil (2001)

[4] A straightforward alternative would be to require that debt at some date does not exceed some level d^*, chosen on the basis of other considerations. In its macroeconomic imbalance procedure, the European Union for example uses a value of d^* of 35% as a trigger to assess whether a member is in a situation of "excess macroeconomic imbalances".

its assets, and is thus able to limit the increase in debt while running a trade deficit. Some emerging market countries are in the opposite situation (see Appendix Table 1). For example, the difference between the average rate of return on liabilities and the average rate of return on assets was 5.2 % for Russia, 3.8% for Malaysia, 2.9% for Thailand; by contrast, the average difference was -1.1% for the United States, -1.7% for Japan.[5][6]

Given often large gross asset and liability positions, these differences in rates can play an important role in debt dynamics. Thus, we extend our earlier condition to take these differences into account. Let A and L denote gross assets and gross liabilities respectively, so that net debt is equal to $D = L - A$. Let r_A and r_L be the rates on gross assets and gross liabilities respectively. Debt dynamics are now given by:

$$L_{t+1} - A_{t+1} = (1 + r_L t)L_t - (1 + r_{At})A_t - NX_t \tag{6}$$

which can be rewritten as:

$$L_{t+1} - A_{t+1} = (1 + r_{Lt})(L_t - A_t) - (r_{At} - r_{Lt})A_t - NX_t \tag{7}$$

Or equivalently

$$D_{t+1} = (1 + r_{Lt})D_t - [NX_t + (r_{At} - r_{Lt})A_t] \tag{8}$$

The relevant flow, in brackets, is now equal to net exports plus the difference in rates times the gross asset position. Dividing by GDP, solving forward as before, and imposing the non-explosion condition gives the following inequality:

$$d_t \leq \sum_{j=0}^{\infty} \prod_{i=0}^{j} \frac{1 + g_{t+i}}{1 + r_{Lt+i}} \, [nx_{t+j} + (r_{At+j} - r_{Lt+j})a_{t+j}] \tag{9}$$

Sustainability requires that the ratio of net debt to GDP be equal to or less than the present value of the ratio of net exports to GDP plus the interest differential times the ratio of gross assets to GDP, denoted by a_t.[7]

[5] These averages are computed over different periods, reflecting data availability. See appendix for dates.

[6] Here again, these facts raise some difficult conceptual issues, whether these differences between average rates of return reflect differences in marginal products or differences in risk or in liquidity across assets. To the extent that they reflect differences in risk, these will be partly reflected in the stochastic simulations we do below when we introduce uncertainty. Countries with more risky assets will experience higher rates of return, but also more uncertainty about the evolution of debt over time. A higher expected rate of return may not imply a higher probability of debt sustainability.

[7] Note that we could have rewritten debt dynamics alternatively as $D_{t+1} = (1 + r_{At})D_t - [NX_t + (r_{At} - r_{Lt})L_t]$, which in turn would have given the following condition:

$$d_t \leq \sum_{j=0}^{\infty} \prod_{i=0}^{j} \frac{1 + g_{t+i}}{1 + r_{At+i}} \, [nx_{t+j} + (r_{At+j} - r_{Lt+j})l_{t+j}]$$

If we apply the same truncation as in the previous section, the inequality becomes:

$$d_t \,(1 - \prod_{i=0}^{n} \frac{1+g_{t+i}}{1+r_{Lt+i}}) \;\leq\; \sum_{j=0}^{n} \prod_{i=0}^{j} \frac{1+g_{t+i}}{1+r_{Lt+i}} \,[(nx_{t+j} + (r_{A\,t+j} - r_{L\,t+j})a_{t+j}] \tag{10}$$

Were all the right hand side variables known for certain, the computation of the index would remain straightforward; the only change is the need to have values for the ratio of gross assets to GDP, a ratio which is not tied down by the debt dynamics condition (a given net debt position may correspond to small or large gross asset and liability positions). But, again, the values of the variables are not known for certain, and this takes us to the next section, about introducing uncertainty.

4 Uncertainty

Neither future net exports, nor growth, nor rates of return on liabilities and assets, are known for certain. As a result the present values above are also random variables. The WEO (or other) forecasts give us the means of the relevant variables but not their distribution. It seems reasonable, in this context, to treat the WEO forecasts as the means of the distribution, but to obtain the higher moments from the historical evidence. Thus, we propose the following approach:

Define the vector $X = [nx, g, r_L, r_A, a]'$, where r_L and r_A are realized rates of return on liabilities and assets, excluding the capital gains from changes in the exchange rate (as the computation of the present discounted value is done conceptually holding the exchange rate constant). Then, one can run the following VAR:[8]

$$\begin{bmatrix} X \\ e \end{bmatrix} = \begin{bmatrix} A_{11}(L) & A_{12}(L) \\ A_{21}(L) & A_{22}(L) \end{bmatrix} \times \begin{bmatrix} X(-1) \\ e(-1) \end{bmatrix} + \begin{bmatrix} \epsilon_X \\ \epsilon_e \end{bmatrix}$$

While the VAR gives the joint movements of all variables including the exchange rate, the forecasts needed to construct the two sides of equation (10) refer to joint movements of the variables in X assuming a constant exchange rate. These can be obtained by using the subsystem (and ignoring the Lucas critique, which may well be relevant in this case):

$$X = A_{11}(L)X(-1) + \epsilon_X \tag{11}$$

where l is the ratio of gross liabilities to GDP. By construction, and despite the fact that the two expressions look rather different, with the present values using different discount factors, they are equivalent.

[8]This assumes that one can construct those adjusted rates of return, and that one has long enough time series and enough degrees of freedom to run the VAR. The two conditions are satisfied for example for the United States, but not necessarily for other countries. We discuss the issue further when looking at Chile below.

The joint distribution of the variables in X, and by implication the joint distribution of the two sides of equation (10) can then be obtained through stochastic simulations of equation (11). Just as before, for each realization, one can compute the exchange rate required to achieve debt sustainability, and obtain the resulting exchange rate distribution.

Before moving on to applications, we want to note the relation of our index to two other measures of external debt sustainability.

The first is the measure used by the IMF as part of the CGER exercise (IMF 2006). It differs from our index in four ways. Three of them are minor. The first is the requirement that debt at some time in the future be equal to debt today, compared to our assumption that properly discounted debt in the future be equal to debt today; when the discount factor is close to 1, which it is in practice, this difference is quantitatively small. The second is in the explicit treatment of rate of return differentials; this is typically not taken into account in the IMF computation, except for a few countries. The third is that the adjustment of the exchange rates affects both the left and right hand side of equation (10); the revaluation of debt, however, is not taken into account in the CGER. The fourth is the most important, namely the explicit treatment of uncertainty. Debt is rarely sustainable for sure, or unsustainable for sure; our index of sustainability gives a more informative signal as to the state of external debt and the need for an adjustment of the exchange rate.

The second is the index proposed by Martin Evans, (Evans 2012) which pays careful attention to rate of return differentials, but uses a different set of approximations from ours in order to get a loglinear approximation to equation (10). Again, the main difference is in our explicit treatment of uncertainty about future returns, net exports, and growth.

5 Application to the United States

We now apply our approach first to the United States in this section and to Chile in the next.

Table 1 gives WEO forecasts for 2017 to 2022, as of the beginning of 2017, together with the asset and liability positions at the end of 2016. [9]

Net debt is forecast to increase from 44% at the end of 2016 to 52% of GDP in 2022. Gross assets are forecast to increase from 129% of GDP at the end of 2016 to 165% of GDP in 2022. Liabilities are forecast to increase from 172% of GDP in 2015 to 217% of GDP in 2022.

The ratio of net exports to GDP is forecast to remain negative, increasing from

[9] Source: March 2017 WEO Database.

3.8% in 2017 to 4.2% in 2022.[10] Real GDP growth is forecast to be higher than the real rate of return on liabilities, implying a discount factor (using r_L as in equation (10) slightly above 1. The rate of return on liabilities is forecast to be lower than the rate of return on assets, reflecting the large foreign holdings of US government bonds on the liability side, and the larger share of FDI and portfolio holdings on the asset side.

Table 1. United States. Data and forecasts, 2016-2022

	2016	2017	2018	2019	2020	2021	2022
Net debt/GDP	44%	44%	46%	47%	49%	51%	52%
Assets/GDP	129%	142%	148%	153%	158%	162%	165%
Liabilities/GDP	172%	186%	194%	200%	207%	213%	217%
Net exports/GDP		-3.8%	-4.1%	-4.4%	-4.4%	-4.2%	-4.2%
Real GDP growth		2.3%	2.5%	2.1%	1.8%	1.7%	1.7%
Real yield on assets		3.6%	3.0%	2.9%	2.7%	2.7%	2.7%
Real yield on liabilities		2.0%	1.9%	1.8%	1.7%	1.6%	1.6%

Suppose, to start, that these forecasts were held with certainty.

The computation of the two terms in equation (10) would then be straightforward. The present discounted value of the truncated sum of net exports plus the yield differential times assets (let us call it the "present value" for short), on the right hand side, is equal to -14.5%. The corresponding value of net debt times one minus the relevant discount factor (let us call it "adjusted net debt" for short), on the left hand side, is equal to -0.6%; note that it is actually negative, despite the fact that US net debt is positive, because the relevant discount factor is slightly above 1. Thus, if we took the forecasts as holding with certainty, we would conclude that debt is not sustainable at the current exchange rate.

The next step would to compute the exchange rate which would make it sustainable. The exchange rate affects both the left and the right-hand side of equation (10):

Starting with the left hand side, computing the effect of the exchange rate on net debt requires knowing the proportion of gross assets and liabilities denominated in foreign currency. In 2012, which is the most recent date for which data appears

[10]We include the secondary income balance (transfers) in net exports. Transfers are, however, small relative to net exports.

to be available (Benetrix et al (2015)), about 68% of US assets were denominated in foreign currency, while only 16% of US liabilities were denominated in foreign currency. Assuming that these proportions remained roughly the same at the end of 2016, and using the numbers in Table 1 for the ratio of assets and liabilities to GDP, implies that a real depreciation of 10% decreases net debt by about 7% of GDP.

Turning to the right hand side of equation and using the semi-elasticity of net exports to the real exchange rate from the IMF's external balance assessment, a real depreciation of 10% increases net exports by about 1.5% of GDP.

Putting the two together, the conclusion would be that, as of the beginning of 2017, and under the assumption that forecasts were held with certainty, debt sustainability would require a real depreciation of about 9.2%.

We now reintroduce uncertainty and follow the approach developed in the previous section. The variables in the VAR are constructed as follows: nx is net exports as a ratio to GDP. g is the growth rate of real GDP. Because the ratio of gross assets and gross liabilities to GDP increases steadily over time, a is defined as the deviation of the ratio of assets to GDP from a linear trend.[11] e is defined as the real effective exchange rate.

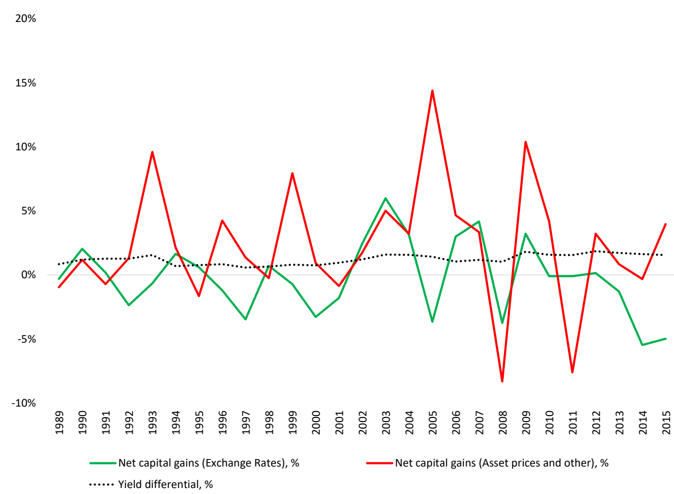
For the rates of return, r_A and r_L, we consider two alternatives. Conceptually, one wants to use the realized real rates of return, including capital gains or losses from changes in asset prices, but not including capital gains or losses due to changes in the exchange rate—as the forecasts are based, both conceptually and empirically (for the WEO forecasts), on the assumption of constant exchange rates. What one can do depends however on data availability. For most countries, one can compute and use real yields on assets and liabilities, defined as nominal yields minus CPI inflation, but ignoring all capital gains and losses.[12] This is our first alternative. For the US, one can go further, and construct real yields plus realized capital gains or losses due to changes in asset prices in their domestic currency.[13] This is our second, and clearly better, alternative. Which alternative is chosen makes a substantial difference to the results. As shown in Figure 1, the variability of capital gains on US assets and liabilities far exceeds that of yields, leading to a much wider distribution of outcomes in simulations below.

[11] The use of the deviation of a from trend is only to get a characterization of the second moments of a. In the construction of the present discounted value of net exports, we use the IMF forecasts of a, which reflect the trend increase in a, as the baseline.

[12] Nominal yields are constructed as the ratio of net investment income (credit or debit) from BOP data, to corresponding assets or liabilities from IIP data. For emerging market countries, data on assets and liabilities are often available only from the mid-1990s or later.

[13] We found similar information for only one other country, Germany.

Figure 1. Real yield and capital gain differentials between US assets and liabilities



We estimate the VAR over 1977-2016 under the first alternative (1976 is the first year for which data on assets and liabilities are available), and over 1989-2015 under the second alternative (1989 and 2015 are the first and latest years for which the decomposition of rates of return is available). The VARs are assumed to be of order 1. The correlation matrices for the innovations $[\epsilon_X, \epsilon_e]$ under both alternatives are shown in Tables 2 and 3. (Diagonal elements denote standard deviations)[14]

Table 2. Correlation matrices of the VAR innovations, using yields

	nx	g	r_L	r_A	a	e
nx	0.005					
g	-0.44	0.014				
rL	-0.13	0.13	0.006			
rA	-0.15	-0.04	0.88	0.006		
a	-0.18	0.19	0.24	0.29	0.064	
e	-0.12	-0.09	0.03	0.16	-0.19	0.063

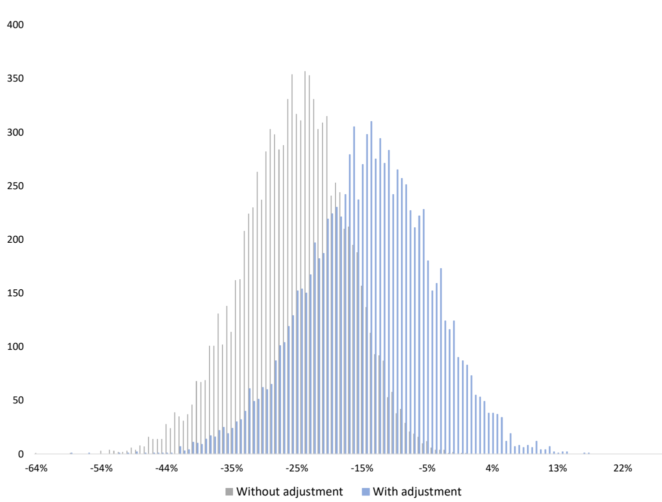
[14]We do not report the matrix of coefficients on lagged variables in the text, but we use equation 11 for our simulations.

Table 3. Correlation matrices of the VAR innovations, with r_A and r_L including capital gains

	nx	g	r_L	r_A	a	e
nx	0.004					
g	-0.81	0.009				
r_L	0.24	-0.008	0.027			
r_A	-0.10	0.29	0.51	0.038		
a	0.09	0.06	0.56	0.63	0.061	
e	-0.15	0.20	-0.42	-0.14	-0.49	0.041

A few correlations stand out: The lower the growth rate, the higher net exports, reflecting the effect of lower output on imports. Rates of return on assets and liabilities excluding capital gains are highly correlated; the correlation is lower when rates of return include capital gains. The tables also show the much larger standard deviations of rates of return including capital gains, 2.7% for r_L and 3.8% for r_A when including capital gains versus 0.6% and 0.6% respectively when they are excluded.

Figure 2a. Present values, excluding or including the term reflecting rate of return differentials



Given the non linearities involved in the present value, rather than solve for the distributions analytically, we use equation (11) to draw 10,000 realizations under the assumption of joint normality of the innovations and use the resulting distributions in what follows.

Figure 2a shows the role of the term reflecting rate of return differentials $(r_A - r_L)\ a$ in the computation of the present value. It plots the distribution of the present value either excluding this term (so that the present value is just the present value of net exports) or including it. In both cases, the rates of return on assets and liabilities are measured by yields, excluding capital gains. Without the term reflecting rate of return differentials, the relevant distribution is the distribution to the left, centered around -26%. With the term reflecting rate of return differentials, the relevant distribution is the distribution to the right, centered around -15%, reflecting the fact that, for the United States, the positive net rate of return differentials offset a large proportion of the trade deficits.

Figure 2b. Present values, excluding or including capital gains.

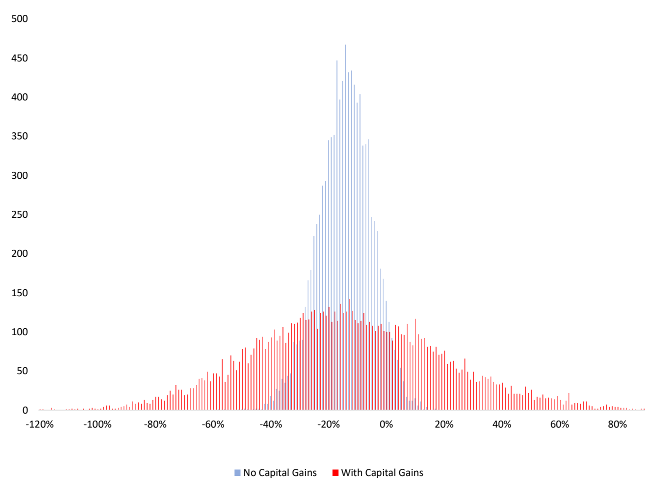


Figure 2b shows the role of the large uncertainty associated with capital gains. It plots the distribution of the present value using either yields, or yields plus capital gains, for r_A and r_L. In both cases, it includes the term reflecting rate of return differentials in the computation of the present discounted value. The means of the distribution are roughly similar, but the striking feature is how much larger the distribution of the present value is when uncertainty about capital gains is taken into account (as it should). The standard deviation of the distribution increases from 9% when using yields to 33% when using yields plus capital gains. We see this result not as a shortcoming of our approach, but as a central fact about the degree of uncertainty about the future, and by implication about the required adjustment of the exchange rate. We return to this issue below.

Figure 3 turns to the left hand side of equation (10) and plots the distribution of the adjusted net debt including or excluding capital gains for r_L (r_A does not appear in the formula for net debt). The means of the two distributions are close,

-0.9% versus -1.2%. The main feature is again the larger distribution when capital gains are taken into account. The standard deviation of the distribution is 5.5% when excluding capital gains versus 6.9% when they are taken into account.

Figure 3. Distribution of adjusted debt

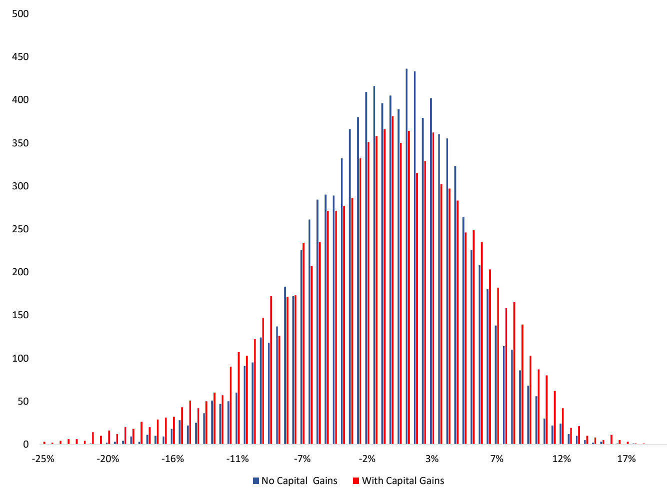
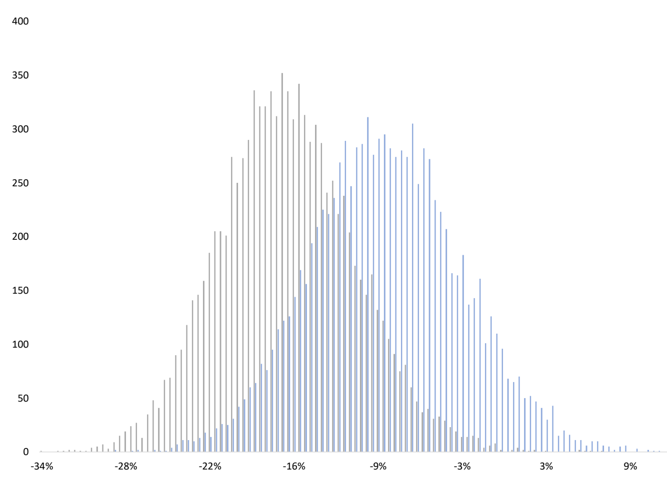


Figure 4a. Distribution of exchange rate adjustments, excluding or including the term reflecting rate of return differentials



Figures 4a and 4b put the two parts together, and look at the implications for the exchange rate. For each simulation, using the same elasticities of debt and net exports to the exchange rate as in the certainty case above, we can compute the exchange rate adjustment such that equation (10) holds as an equality, and report the distribution.

Figure 4a parallels Figure 2a, showing the role of the term reflecting rate of return differentials $(r_A - r_L)\,a$ in the computation of the present value. It plots the distribution of the exchange rate adjustments either excluding this term from the present value (so that the present value is just the present value of net exports) or including it. In both cases, the rates of return on assets and liabilities are measured by yields, excluding capital gains. Including rate of return differentials moves the distribution to the right, implying a smaller decrease in the exchange rate. The mean of the distribution is -9.1%, compared to a mean of -16.4% when rate of return differentials are excluded. The standard deviations of the two distributions are rather similar, 5.6% and 5.1%.

Figure 4b. Distribution of exchange rate adjustments, excluding or including capital gains

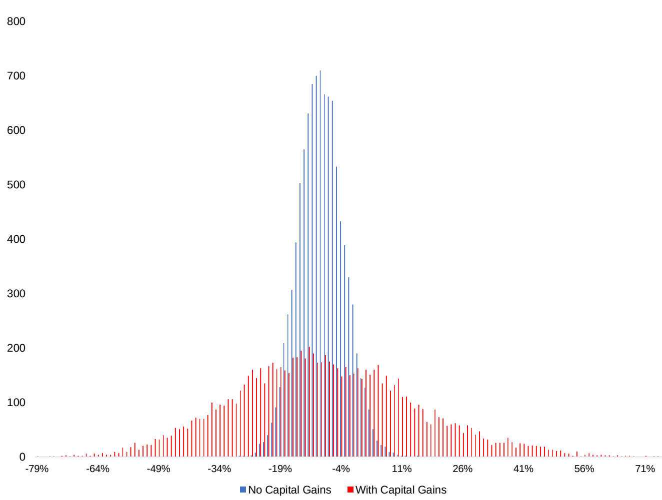


Figure 4b shows the role of uncertainty associated with capital gains. It plots the distribution of exchange rate adjustments excluding or including capital gains. In both cases, it includes the term reflecting rate of return differentials in the computation of the present discounted value. The means of the distribution are roughly similar, -9.1% and -8.2% (they would be the same is the present value was linear in the random variables), but the striking feature is again how much larger the distribution of the present value is when uncertainty about capital gains is taken into account. The standard deviation of the distribution is 5.6% when capital gains are excluded versus 22.6% when they are taken into account. The probability that the exchange rate has to depreciate is nearly 100% when excluding capital gains, but only 64% when they are included.

To summarize, we have shown how our index of sustainability can be constructed using US data. Our conclusion is that, looking at the US economy as of the beginning of 2017, and assuming forecasts are held with certainty, the mean exchange rate depreciation needed to maintain debt sustainability is about 8.3%. But, because of the uncertainty associated with net exports and rates of return, the distribution of exchange rate adjustments has a large support. The probability that debt is actually sustainable at the current exchange rate is fairly high, equal in our computation to 36%.

6 Application to Chile

We now use the same methodology to look at an emerging market country, namely Chile.

Table 3. Chile: Data and forecasts, 2016-2022

	2016	2017	2018	2019	2020	2021	2022
Net Debt/GDP	20%	21%	22%	23%	24%	25%	25%
Assets/GDP	134%	139%	140%	141%	142%	143%	142%
Liabilities/GDP	154%	159%	162%	164%	166%	167%	167%
Net exports/GDP		2.4%	2.2%	2.1%	2.0%	1.9%	1.9%
Real GDP growth		1.7%	2.3%	2.7%	2.9%	3.1%	3.2%
Real yield on assets		1.1%	1.0%	0.9%	0.8%	0.6%	0.5%
Real yield on liabilities		3.4%	3.3%	3.2%	3.2%	3.1%	3.0%

Table 3 gives WEO forecasts for 2017 to 2022, as of the beginning of 2017, together with the asset and liability positions at the end of 2016.

Net debt is forecast to increase from 20% of GDP at the end of 2016 to 25% of GDP by 2022. Gross assets are forecast to increase from 134% of GDP at the end of 2016 to 142% of GDP in 2022. Liabilities are forecast to increase from 154% of GDP at the end of 2016 to 167% of GDP in 2022.

The ratio of net exports to GDP is forecast to be positive, around 2%. In contrast to the United States, the rate of return on liabilities is forecast to be higher than the rate of return on assets, reflecting the large foreign holdings of US government bonds on the asset side, and the larger share of FDI and portfolio holdings on the liability side.

Suppose again that we held these forecasts with certainty. Going through the same steps as before, the present value is equal to -7.5%, reflecting the fact that the projected trade surpluses are not sufficient to offset the higher rateof return on liabilities than on assets. The value of adjusted debt is small, 0.6%, reflecting both the fact that net debt is small and the discount factor is close to 1. A negative present value and a positive adjusted debt both imply that the condition given by equation (10) is not satisfied at the current exchange rate.

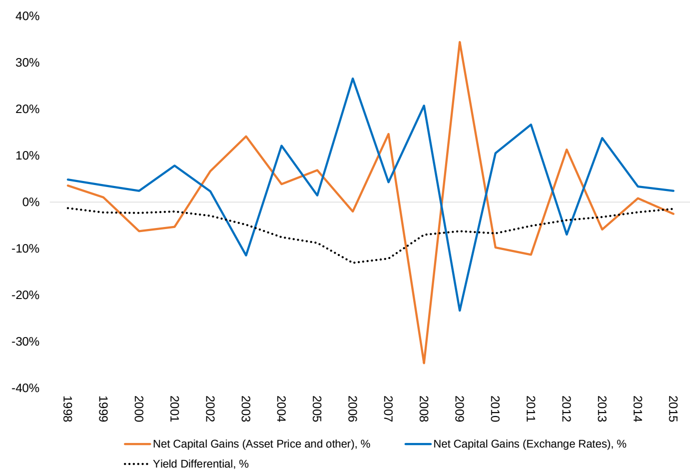
The next step is to compute the exchange rate that would satisfy that condition. Computing the effect of the exchange rate on net debt requires knowing the proportion of gross assets and liabilities denominated in foreign currency. As of 2012 (data from Benetrix et al (2015)), nearly all foreign assets were denominated in foreign currency. About 30% of foreign liabilities were denominated in foreign currency. Assuming that the proportions have not changed since, and using the numbers in Table 3 for assets and liabilities, this suggests that a depreciation of the peso of 10% improves the net debt position of Chile by roughly 8.8% of GDP. Turning to the present value and using the semi-elasticity of net exports to the real exchange rate from the IMF's external balance assessment, a real depreciation of 10% increases net exports by about 3.0% of GDP. Putting the two together, under the assumption that forecasts were held with certrainty, the conclusion would be that, as of the beginning of 2017, debt sustainability required a real depreciation of about 3.2%.

We now reintroduce uncertainty. While data on yields on assets and liabilities are available, series for rates of return, including capital gains on asset prices but excluding capital gains coming from movements in exchange rates are not available. And, as we saw in the previous section, much of the uncertainty about the future comes from capital gains, and this uncertainty has a large effect on the distribution of exchange rate adjustments. Thus, we must first construct the relevant series.

We first obtain realized rates of return by computing the change in the value of assets and liabilities, adjusted for outflows and inflows respectively. Then, using the approach proposed by Benetrix et al (2015), we compute the change in the value of assets due to exchange rate movements, and do the same thing for liabilities. We then subtract these valuation effects from the realized rates of return. (See Appendix 2).

Figure 5 shows net capital gains from exchange rate changes (in blue) and those from changes in asset prices (in orange) since 1998. As with the United States, the variability of net capital gains is significantly larger than that of yields, which will be reflected in the wider distributions in the simulations below. Note the large deviations between capital gains from exchange rates and asset prices in 2008 and 2009. In 2008, the large depreciation of the peso caused large capital gains from exchange rate changes while asset price declines caused large capital losses. The opposite was observed in 2009, reflecting the large appreciation of the peso and a recovery of asset prices.

Figure 5. Real yield and capital gains differentials on assets versus liabilities, Chile



The next step is to construct the VAR. Because we have a short sample for some of the variables, we run two restricted VARs. To capture the correlation between net exports and growth, we run a VAR of order 1 in nx, g, using data from 1980 on. To capture the correlation between the rates of return on assets and on liabilities, we run a VAR of order 1 in r_L, r_A, a, e, using data from 1998 on. (One may want to keep 2008 and 2009 or leave them out on the assumption that they were sui generis, and a similar crisis is unlikely in the future. We leave them out. Leaving them in increases the standard deviations of the residuals for r_L and r_A but does not make a major difference to the conclusions below.) We then use the covariance matrix of the innovations from the two VARs for our simulations.[15]

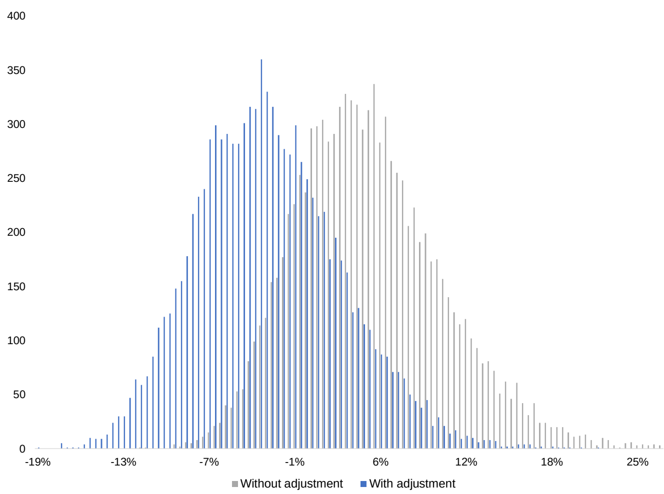
The correlation matrix is given in Table 4. Note the large negative correlation between nx and g innovations, reflecting the effect of output on imports. Note also that the standard deviations of the residuals for r_A and r_L are about 50% larger than those for the United States

[15]This amounts to running one VAR with zero restrictions on some of the off-diagonal elements of the matrix A_{11}.

Table 4. Correlation matrix of the VAR innovations. Real yields including capital gains

	nx	g	r_L	r_A	a	e
nx	0.029					
g	-0.13	0.032				
rL	0.35	-0.35	0.046			
rA	0.03	-0.15	0.002	0.053		
a	0.18	-0.18	-0.12	0.86	0.054	
e	-0.15	0.09	-0.10	-0.56	-0.74	0.038

Figure 6a. Distribution of exchange rate adjustments excluding or including the term reflecting rate of return differentials, Chile



We can then go through the same steps as we did for the United States. We just present the two final figures giving the distribution of the exchange rate adjustments, corresponding to figures 4a and 4b for the United States earlier.

Figure 6a shows the role of the term reflecting rate of return differentials $(r_A - r_L)a$ in the computation of the present value. It plots the distribution of the exchange rate adjustments either excluding this term from the present value (so that the present value is just the present value of net exports) or including it. In both cases, the rates of return on assets and liabilities are measured by yields, excluding capital gains. In contrast to the United States, including rate of return differentials moves the distribution to the left, implying a larger decrease in the exchange rate. The mean of the distribution is an appreciation of 4.2% when rates of return differentials are excluded, compared to a depreciation of 2.7% when rate of return differentials are included. The difference reflects the fact that Chile has large gross asset and liability positions, and pays a higher rate on average on its liabilities than it receives on its assets.

Figure 6b. Distribution of exchange rate adjustments excluding or including capital gains , Chile

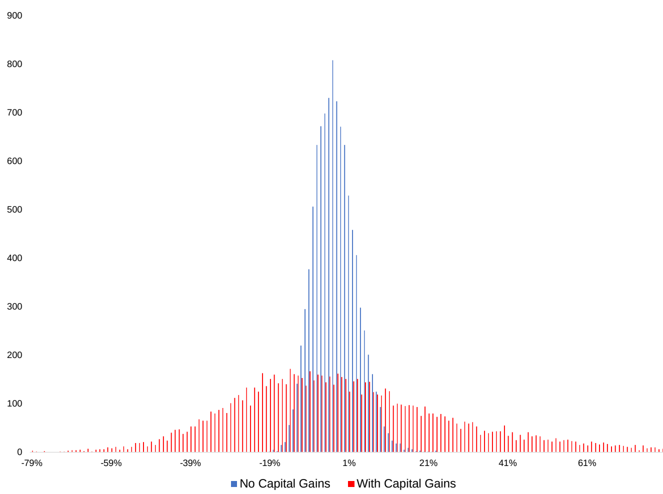


Figure 6b shows the role of uncertainty associated with capital gains. It plots the distribution of exchange rate adjustments excluding or including capital gains for r_A and r_L. In both cases, it includes the term reflecting rate of return differentials in the computation of the present discounted value. The means of the distribution are close, -2.7% and 0.9%, but the striking feature is again how much larger the distribution of the present value is when uncertainty about capital gains is taken into account. The standard deviation of the distribution is 5.3% when excluding capital gains versus 31.0% when they are taken into account. The probability that the exchange rate has to depreciate is 69% when capital gains are not taken into account, but 49% when they are (reflecting the fact that the support of the distribution is so wide as to make it nearly uninformative).

We draw two conclusions from this examination of Chile, and they parallel those obtained for the United States:

First, just looking at trade balances may be misleading and one has to look at rate of return differentials. In the case of Chile, as opposed to the case of the United States, negative rate of return differentials yield a more negative assessment of the required exchange rate adjustment.

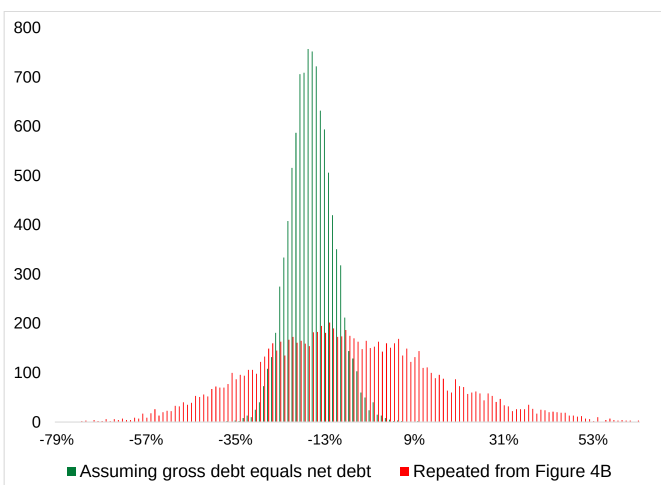
Second, and to our mind, most importantly, the uncertainty associated with realized rates of return implies a very large distribution of required exchange rate adjustments. Even if forecasts imply that, ignoring uncertainty, debt is unsustainable at the current exchange rate, the probability that a downward adjustment of the exchange rate will be required can be substantially less than one. In the case of Chile, while baseline numbers suggest the need for a (small) depreciation, the probability that the exchange rate has to depreciate is only 49%.

7 Remarks and conclusions

Our paper makes two contributions.

The first is methodological. We propose a new index of debt sustainability, namely the probability that debt is sustainable at the current exchange rate. We show how to take into account both the role of differential rates of return on assets and liabilities, as well as the implications of uncertainty, in determining this probability.

Figure 7. Distribution of exchange rate adjustments. Actual and counterfactual, assuming gross liabilities equal net liabilities. United States.



The second is empirical. We show how much both dimensions matter. One major fact about today's world is the size of the gross liability and asset positions relative to the net debt position. These, together with large variations in rates of return on these assets and liabilities, imply substantial uncertainty about whether debt is sustainable or not. This is best illustrated by considering a counterfactual experiment, in which we assume that U.S. gross and net liability positions are the same. Specifically, we assume that US gross liabilities are 44% of GDP, gross assets are equal to zero, and so net debt is 44%, as it is in Table 1. The counterfactual distribution of exchange rate adjustments is given by the distribution in green in Figure 7. The actual distribution, repeated from Figure 4b, is shown in red. It shows the dramatic increase in the support of the distribution going from the counterfactual to the actual distribution, the difference being due to the size of the gross positions relative to the net position. In the counterfactual distribution, the probability that the exchange rate must depreciate is nearly 100%. Given the large gross positions, the probability decreases to 64%.

We have stayed away from policy implications. Our results make it clear that debt sustainability depends much more on shocks to the capital account than to shocks to the current account.[16] Given a ratio of assets to GDP of nearly 150% as in the case of Chile, a one-standard deviation shock to the rate of return on assets of 5.3% moves debt by 8% of GDP, overwhelming likely movements in the trade balance. Put another way, dramatic measures taken to adjust the trade balance may only move the probabilities a bit, and any adjustment in the trade balance can easily be offset by capital gains or losses on gross assets and liabilities. This does not mean that governments should ignore trade deficits, but they should be aware that trade balance adjustments, which maybe potentially painful, may well be swamped by capital account shocks.

[16] This echoes the theme developed by Obstfeld (2012).

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8 Appendix 1. Discount rates and discount factors

		rA	rL	Discount rates using rA	Discount rates using rA
Australia	1989-2015	4.0%	5.1%	0.99	0.98
Belgium	1996-2015	4.0%	4.0%	0.98	0.98
Brazil	2002-2015	2.7%	5.4%	1.00	0.98
Canada	1991-2015	3.3%	4.6%	0.99	0.98
China	2005-2015	3.8%	6.9%	1.05	1.02
France	1981-2015	4.5%	3.9%	0.97	0.98
Germany	1989-2015	3.2%	3.3%	0.98	0.98
India	1997-2015	3.7%	4.8%	1.03	1.02
Indonesia	2002-2015	3.7%	4.8%	1.02	0.98
Italy	1999-2015	4.0%	3.7%	0.96	0.97
Japan	1996-2015	3.5%	1.8%	0.97	0.99
Malaysia	1981-2015	5.7%	9.4%	1.00	0.97
Mexico	2002-2015	2.5%	4.1%	1.00	0.98
Netherlands	1989-2015	4.9%	4.9%	0.97	0.97
Poland	1995-2015	7.2%	5.5%	0.97	0.99
Russia	2000-2015	4.2%	9.4%	1.00	0.95
South Africa	1989-2015	8.4%	6.6%	0.95	0.96
Spain	1989-2015	5.4%	4.6%	0.97	0.98
Thailand	1999-2015	4.1%	6.9%	1.00	0.97
Turkey	1997-2015	3.9%	3.6%	1.01	1.01
United Kingdom	1989-2015	4.4%	4.3%	0.98	0.98
United States	1979-2015	5.8%	4.7%	0.98	0.99

Discount rates are computed as the average of the ratio of real GDP growth to either the real yield on assets or the real yield on liabilities. All variables are in local currency. Discount factors are computed as one over one plus the discount rate.

9 Appendix 2

Following Benetrix (2012), this appendix shows how to approximate realized returns on foreign assets and foreign liabilities excluding capital gains from movements in exchange rates. The steps are described for returns on foreign assets; they are the same for foreign liabilities.

Realized rates of returns (yield plus capital gains) on foreign assets are given by:

$$r_{At} = (IA_t + KGA_t)/A_{t-1}$$

where IA and KGA are respectively the investment income and capital gains realized on the existing stock of assets. Capital gains realized between the end of year $t-1$ and the end of year t are given by:

$$KGA_t = A_t - A_{t-1} - KO_t$$

where KO_t are capital outflows (or net acquisition of assets) during t. All variables are measured in local currency.

Realized capital gains on foreign assets reflect both changes in asset prices, KGA^P, and changes in exchange rates, KGA^E. We compute KGA^E as follows:

Let the share of assets denominated in foreign currency i at the end of year t be denoted by ω_{it}. And let $\alpha_t = \sum_{i=1}^{n} \omega_{it}$ be the overall share of assets denominated in foreign currency.

Denote by Δe_{it} the rate of change of the ith bilateral exchange rate from the end of year $t-1$ to the end of year t ($\Delta e > 0$ is a depreciation), and define the weighted rate of change of the exchange rate as:

$$\Delta e_t = \sum_{i=1}^{n} \omega_{it-1} \, \Delta e_{it}$$

We then approximate KGA_t^E by $KGA_t^E = \alpha_{t-1} \, A_{t-1} \, \Delta e_t$, and subtract it from KGA_t to obtain KGA_t^P.

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